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16. Abstract

A solution of the nonstationary transport equation for the case of a semi-infinite amisotropic medium with a severely elongated scattering envelope is possible.

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NONSTATIONARY RADIATION FIELD IN A SEMI-INFINITE MEDIUM WITH NONISOTROPIC SCATTERING

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In conjunction with practical requirements, a nonstationary radiation field theory has recently been developed successfully. The theory has extremely wide application in areas of endeavor using pulsed light sources for purposes of rangefinding, communications, atmospheric and water basin probes, etc. The solution of the nonstationary transport equation in an isotropically scattering semi-infinite medium by the Schuster-Schwarzschild method is given in [1]. However, the solution obtained is not applicable for calculating light fields in anisotropic media with a severely elongated light-scattering envelope. In this paper it is solved approximately by V.V. Sobolev's method [2], in which first-order scattering is taken into consideration exactly, and higher-order scattering is treated approximately, by retaining only the first two terms in the Legendre polynomial expansion of the equation for the envelope. However, the use of this method in the transition to the nonstationary problem leads to extremely complicated formulas, so that for simplicity we assume the higher-order scattering is isotropic. /70*

We will present briefly the solution of the stationary problem, and then we will extend it to the nonstationary case.

Let the boundary of a semi-infinite isotropically scattering medium be illuminated by parallel rays, incident at an angle arccos ζ. Let us denote by πS the illuminance of a unit area, perpendicular to the rays at the boundary of the medium. Let us ignore the effect of light transmission through the boundary, i.e., we will assume that the radiation enters the medium. In the case of isotropic scattering the radiation transport and the radiant equilibrium equation have the form /71

$$\eta \frac{dI(\tau, \eta, \zeta)}{d\tau} = -I(\tau, \eta, \zeta) + B(\tau, \eta, \zeta); \quad (1)$$

$$B(\tau, \eta, \zeta) = \frac{\lambda}{2} \int_{-1}^{1} I(\tau, \eta, \zeta)\, d\eta + \frac{\lambda}{4} S e^{-\tau/\zeta}. \quad (2)$$

*Numbers in the margin indicate pagination in the foreign text.

Here τ is the optical depth; $I(\tau, \eta, \zeta)$ is the intensity of the diffusely scattered radiation, traveling at an angle arccos η to the normal; $B(\tau, \eta, \zeta)$ is the source function.

Let us denote by $I_1(\tau, \zeta)$ and $I_2(\tau, \zeta)$ the intensities of the light traveling inside the medium and in the opposite direction, i.e., let us state

$$I_1(\tau, \zeta)=\int_0^1 I(\tau, \eta, \zeta)\, d\eta; \tag{3}$$

$$I_2(\tau, \zeta)=\int_{-1}^0 I(\tau, \eta, \zeta)\, d\eta. \tag{4}$$

In the Schuster-Schwarzschild approximation Eqs. (1) and (2) with the notations of (3) and (4) are reduced to the form

$$\frac{d^2(I_1+I_2)}{d\tau^2}-k^2(I_1+I_2)+2\lambda S e^{-\tau/\zeta}=0, \tag{5}$$

where

$$k^2=4(1-\lambda). \tag{6}$$

The solution of Eq. (5) must be sought for the boundary condition that takes into consideration that diffuse radiation, traveling inside the medium, is absent at the boundary, i.e.,

$$I_1(\tau, \zeta)=0 \quad \text{for} \quad \tau=0. \tag{7}$$

The solution of Eq. (5) has the form

$$I_1+I_2=Ce^{-k\tau}-De^{-\tau/\zeta}, \tag{8}$$

where

$$D=\frac{2\lambda\zeta^2}{1-k^2\zeta^2}S. \tag{9}$$

For the boundary condition (7)

$$C = D\frac{1}{\zeta}\frac{1+2\zeta}{2+k}. \tag{10}$$

from which we obtain the expression /72

$$B(\tau,\zeta) = \frac{\lambda\zeta(1+2\zeta)}{(2+k)(1-k^2\zeta^2)}Se^{-k\tau} + \frac{\lambda}{4}\frac{1-4\zeta^2}{1-k^2\zeta^2}Se^{-\tau/\zeta}. \tag{11}$$

for the source function $B(\tau, \zeta)$. For light normally incident on the boundary of the medium the last formula assumes the form

$$B(\tau, 1) = \frac{3\lambda^2}{(2+k)(1-k^2)}Se^{-k\tau} - \frac{3}{4}\frac{\lambda}{1-k^2}Se^{-\tau}. \tag{12}$$

According to what has been said above, to calculate $B(\tau, \zeta)$ for an arbitrary scattering function the term, taking account of the first-order isotropic scattering, must be subtracted from the right side of Eq. (11) and a term, corresponding to the given function, must be added. As a result we arrive at the expression

$$B(\tau,\zeta) = \frac{\lambda\zeta(1+2\zeta)}{(2+k)(1-k^2\zeta^2)}Se^{-k\tau} + \frac{\lambda}{4}S\left[x(\gamma) - \frac{4\lambda\zeta^2}{1-k^2\zeta^2}\right]e^{-\tau/\zeta}, \tag{13}$$

and for $\zeta = 1$

$$B(\tau, 1) = \frac{3\lambda^2}{(2+k)(1-k^2)}Se^{-k\tau} + \frac{\lambda}{4}S\left[x(\gamma) - \frac{4\lambda}{1-k^2}\right]e^{-\tau}. \tag{14}$$

Let us go on to the nonstationary case. Let us denote by t_1 and t_2 the average time spent by a quantum in the absorbed state and in transit between two successive scattering events, respectively. In place of the time t we introduce the dimensionless time u and the constants β_1 and β_2:

$$u = \frac{t}{t_1+t_2}; \quad \beta_1 = \frac{t_1}{t_1+t_2}; \quad \beta_2 = \frac{t_2}{t_1+t_2}. \tag{15}$$

Usually β_1 and β_2 are different in their order of magnitude. Since in problems of atmospheric optics and hydro-optics nonstationarity is caused primarily by the finiteness of the velocity of light, then $\beta_1 << \beta_2$; therefore below let us take $\beta_1 = 0$, $\beta_2 = 1$.

Let us assume that S(u) = δ (u), where δ (u) is the Dirac delta function, i.e., we will assume that the medium was subjected to an instantaneous action of radiation sources. To obtain the solution in the nonstationary case we use a known method [3], which is based on finding the Laplace transform of any characteristic of the radiation field, with respect to time directly from the corresponding characteristic of the stationary process. To do this λ in the solution of the stationary process must be replaced by $\lambda/((1 + \beta_1 s)(1 + \beta_2 s))$ and τ by $\tau(1 + \beta_2 s)$. Here s is the parameter of the Laplace transform.

As a result we obtain /73

$$\overline{B}(s, \tau, \zeta) = \frac{\lambda^2 \zeta (1+2\zeta)}{2(1-4\zeta^2)} \times$$
$$\times \frac{e^{-2\tau\sqrt{(1+s)(1+s-\lambda)}}}{\left(1+s+\frac{4\lambda\zeta^2}{1-4\zeta^2}\right)\left(1+s+\sqrt{(1+s)(1+s-\lambda)}\right)} +$$
$$+ \frac{\lambda}{4(1+s)}\left[x(\gamma) - \frac{4\lambda\zeta^2}{(1-4\zeta^2)\left(1+s+\frac{4\lambda\zeta^2}{1-4\zeta^2}\right)}\right] e^{-\tau(1+s)/\zeta} \tag{16}$$

and

$$\overline{B}(s, \tau, 1) = -\frac{\lambda^2}{2\left(1+s-\frac{4}{3}\lambda\right)} \frac{e^{-2\tau\sqrt{(1+s)(1+s-\lambda)}}}{1+s+\sqrt{(1+s)(1+s-\lambda)}} +$$
$$+ \frac{\lambda}{4(1+s)}\left[x(\gamma) + \frac{4\lambda}{3\left(1+s-\frac{4}{3}\lambda\right)}\right] e^{-\tau(1+s)}. \tag{17}$$

As is known, the intensity of the diffuse light, at an optical depth τ, is determined by the formulas

$$I(\tau, \eta, \zeta) = \int_0^\tau B(\tau', \eta, \zeta) e^{-\frac{\tau-\tau'}{\eta}} \frac{d\tau'}{\eta} \quad \text{for } \eta \geqslant 0; \tag{18}$$

$$I(\tau, \eta, \zeta)=\int_{\tau}^{\infty} B(\tau', \eta, \zeta) e^{-\frac{\tau'-\tau}{\eta}} \frac{d\tau'}{\eta} \quad \text{for} \quad \eta \leqslant 0. \tag{19}$$

In the nonstationary case for $\zeta = 1$ we obtain

$$\bar{I}(s, \tau, \eta)=\frac{\lambda}{4(1-\eta)(1+s)}\left[x(\gamma)+\frac{4\lambda}{3\left(1+s-\frac{4}{3}\lambda\right)}\right]\times$$
$$\times\{e^{-\tau(1+s)}-e^{-\tau(1+s)/\eta}\}-\frac{\lambda^2}{2(1-2\eta)\left(1+s-\frac{4}{3}\lambda\right)}\times$$
$$\times\frac{e^{-2\tau\sqrt{(1+s)(1+s-\lambda)}}-e^{-\tau(1+s)\eta}}{1+s+\sqrt{(1+s)(1+s-\lambda)}+\frac{2\lambda\eta}{1-2\eta}} \tag{20}$$

for $\eta \geqslant 0$ and

$$\bar{I}(s, \tau, \eta)=\frac{\lambda}{4(1+\eta)(1+s)}\left[x(\gamma)+\frac{4\lambda}{3\left(1+s-\frac{4}{3}\lambda\right)}\right]e^{-\tau(1+s)}-$$
$$-\frac{\lambda^2}{2(1+2\eta)\left(1+s-\frac{4}{3}\lambda\right)}\frac{e^{-2\tau\sqrt{(1+s)(1+s-\lambda)}}}{1+s+\sqrt{(1+s)(1+s-\lambda)}+\frac{2\lambda\eta}{1+2\eta}} \tag{21}$$

for $\eta \leqslant 0$.

Manipulation of Eqs. (20) and (21) yields (see [4])

$$I(\tau, \eta, u)=[f_1(\tau, \eta, u)+f_5(\tau, \eta, u)]\,\Delta(u-\tau)-[f_1(\tau, \eta, u)+$$
$$+f_2(\tau, \eta, u)-f_3(\tau, \eta, u)]\,\Delta(u-2\tau)-[f_3(\tau, \eta, u)-f_4(\tau, \eta, u)+$$
$$+f_5(\tau, \eta, u)]\,\Delta\left(u-\frac{\tau}{\eta}\right) \tag{22}$$

for $\eta \geqslant 0$;

$$I(\tau, \eta, u)=[f_1(\tau, \eta, u)+f_5(\tau, \eta, u)]\,\Delta(u-\tau)-$$
$$-[f_1(\tau, \eta, u)+f_2(\tau, \eta, u]\,\Delta(u-2\tau) \tag{23}$$

for $\eta \leqslant 0$. Here

$$f_1(\tau, \eta, u) = \frac{\lambda}{4} \frac{e^{\frac{3}{4}\lambda(u-\tau)-u}}{1-\eta}; \tag{24}$$

$$f_2(\tau, \eta, u) = \frac{\lambda}{2\pi}(1-2\eta)\, e^{-\left(1-\frac{\lambda}{2}\right)u} \int\limits_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} \times$$

$$\times \frac{\left(t+\frac{\lambda}{2}\frac{1+2\eta}{1-2\eta}\right)\sin\tau\sqrt{\lambda^2-4t^2} + \frac{\sqrt{\lambda^2-4t^2}}{2}\cos\sqrt{\lambda^2-4t^2}}{\left(t-\frac{5}{6}\lambda\right)\left[t\,(1-4\eta^2)+\frac{\lambda}{2}(1+4\eta^2)\right]}\, e^{tu}\, dt; \tag{25}$$

$$f_3(\tau, \eta, u) = \frac{3}{2} \frac{\lambda\eta}{(1+2\eta)(1-\eta^2)}\, e^{\left\{\frac{4\lambda\eta}{1-4\eta^2}\tau - \left[1+\frac{4\lambda\mu^2}{1-4\eta^2}\right]u\right\}}; \tag{26}$$

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$$f_4(\tau, \eta, u) = \frac{\lambda}{4\pi}(1-2\eta)\, e^{-\left(1-\frac{\lambda}{2}\right)u} \int\limits_{-\frac{\lambda}{2}}^{\frac{\lambda}{2}} \times$$

$$\times \frac{\sqrt{\lambda^2-4t^2}\, e^{t(u-\tau/\eta)}}{\left(t-\frac{5}{6}\lambda\right)\left[(1-4\eta^2)+\frac{\lambda}{2}(1+4\eta^2)\right]}\, dt; \tag{27}$$

$$f_5(\tau, \eta, u) = \frac{\lambda}{4(1-\eta)}\left[x(\gamma)-1\right] e^{-u}; \tag{28}$$

$$\Delta(x) = \begin{cases} 1 & \text{for } x \geqslant 0 \\ 0 & \text{for } x < 0. \end{cases} \tag{29}$$

It should be noted that for $x(\gamma) = 1$ Eqs. (22)–(27) are identical to the results obtained in [1].

Graphs of the temporal and angular structure of the light field were calculated from our formulas. Figure 1 shows curves of the spreading of the δ-pulse of light at an optical depth of $\tau = 1$ in an isotropically scattering medium for forward directions of the diffuse radiation, at a 45° angle, and backward (180°); in addition, Fig. 1 shows a similar curve for the backward direction of scattered light in a medium having a scattering indicatrix of $x(\gamma) = 1 + x_2\cos\gamma$ for $x_1 = 0.9$. As seen from Fig. 1, the nature of the variation of the curves matches the results of direct measurements and calculations by the Monte Carlo method [5]. In the case of nonisotropic scattering the signal, reflected by the medium, has a maximum, which is displaced to the right with an increase in the optical depth, and the curve itself becomes steeper.

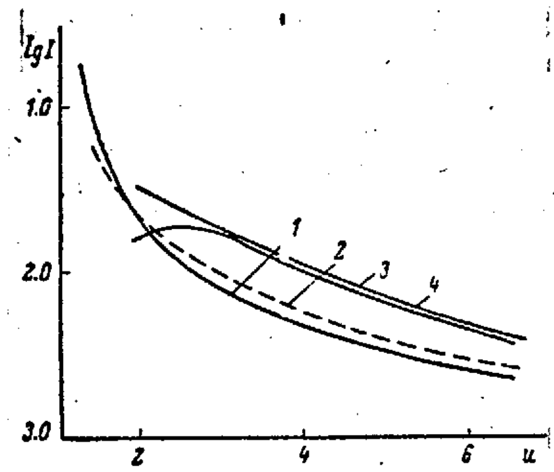

Fig. 1. Curves of the spreading of a δ-pulse at an optical depth of $\tau \leqq 1$. θ: 1) 0; 2) 45°; 3) 180°; 4) 180° in a medium with a scattering function $x(\gamma) = 1 + 0.9\cos\gamma$. /76

The angular structure of the light field in an isotropic and a nonisotropic medium is shown in Fig. 2. Here the intensity of the diffusely scattered radiation is plotted in polar coordinates as a function of the scattering angle at the instants of dimensionless time $u = 2$ and $u = 6$. All the curves are normalized to the corresponding value of the intensity of the backward scattered light. The dashed lines denote the curves obtained for an isotropic medium, copied by us from [1]; the continuous lines are for a nonisotropic medium with scattering functions of $x(\gamma) = 1 + 0.9\cos\gamma$. As seen from the figure, with the changeover to a nonspherical scattering function the fraction of light, scattered forward and to the side, is increased; but the nature of the curves is not altered. A jump in the intensity value is visible on all the curves for the angles arccos τ/u. As pointed out in [1], the jump is explained by the fact that at this instant of time first-order scattered radiation disappears. The size of the jump decreases rapidly with an increase in optical depth, and for $\tau \geqslant 10$ it is nearly absent. This result occurs both for isotropic as well as for nonisotropic scattering. From the formulas (22)-(29) it is easy to obtain similar results for real, severely elongated envelopes too.

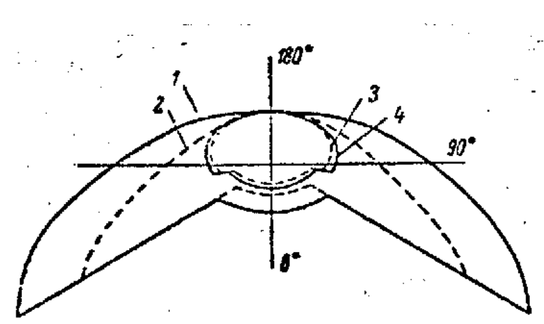

Fig. 2. Angular structure of the light field at an optical depth of $\tau = 1$. 1) $u = 2$, $x(\gamma) = 1 + 0.9\cos\gamma$; 2) $u = 2$, $x(\gamma) = 1$; 3) $u = 6$, $x(\gamma) = 1$; 4) $u = 6$, $x(\gamma) = 1 + 0.9\cos\gamma$.

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